used effectively in the mode of the method of trials when the electronic computer has a display. A new kind of "resolver," whose main elements are an electronic computer performing the most routine part of the work, a display which permits operational analysis and decision making, and an operator-calculator which forms a new model for approbation on the basis of the data obtained, hence originates.

LITERATURE CITED

1. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Incorrect Problems [in Russian], Nauka, Moscow (1974).
2. V. A. Morozov, Regular Methods of Solving Incorrectly Formulated Problems [in Russian], Izd. Mosk. Gos. Univ. (1974).
3. V. A. Morozov and N. L. Gol'dman, Numerical Analysis by FORTRAN [in Russian], Izd. Mosk. Gos. Univ. (1976), No. 15, p. 52.
4. A. V. Goncharskii and A. G. Yagola, Dokl. Akad. Nauk SSSR, 184, No. 4 (1969).
5. M. K. Samarin, Numerical Analysis by FORTRAN [in Russian], Izd. Mosk. Gos. Univ (1974), No. 7, p. 101.
6. M. K. Samarin, Numerical Analysis by FORTRAN [in Russian], Izd. Mosk. Gos. Univ. (1976), No. 15, p. 83.
7. V. K. Ivanov, Dokl. Akad. Nauk SSSR, 145, No. 2 (1962).

SOLUTION OF INVERSE HEAT-CONDUCTION PROBLEMS ON SPECIALIZED ANALOG COMPUTERS

M. P. Kuz'min, I. M. Lagun, and S. V. Lagun

UDC 536.24

Recommendations on the application of specialized analog computers for the solution of inverse problems of heat conduction are given. The presence of a zone of sensitivity delimiting the possible location of a primary information source is established.

Inverse problems are quite extensive in heat- and mass-transfer processes. This is explained primarily by the fact that measurement of the parameters of these processes (temperature, heat-flux density, etc., for instance) in the range of high values under non-steady-state conditions is difficult, and a completely insurmountable problem in a number of cases. In such situations inverse problems are the most acceptable method of solving these problems.

Inverse problems of heat conduction are used in thermal power plants to establish the thermal gasdynamic circumstances according to the results of temperature measurements, to determine uniqueness conditions, and for machine design. In connection with the growing heat loads, the determination of the thermal environment in the high-temperature range, i.e., the heat-flux density q_S and the surface temperature T_S, the temperature of the gas stream T_g flowing around a solid, the coefficient of heat transfer between the hot gas stream and the solid α_g, etc., according to the results of temperature measurements in the low-temperature range, is a problem which must be solved in engineering. Inverse problems of heat conduction are important in the design and construction of heat shields, in the prediction of the thermophysical properties of materials with a given operating range, etc.

If the process of heat transfer between a medium and a solid is considered, then depending on the location of the quantity to be determined inverse problems of heat conduction can be separated into three classes: internal, external, and combined. We shall refer such problems for which the parameters (characteristics) within the body or on its surfaces are determined as a result of the solution to internal, problems when the characteristics of the environment are found to external, and problems for which combinations of parameters of the first two classes will be the subject of solution to the combined classes. A diagram of the classification of inverse problems of heat conduction is shown in Fig. 1.

Tul'skii Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1123-1130, December, 1977. Original article submitted April 5, 1977.

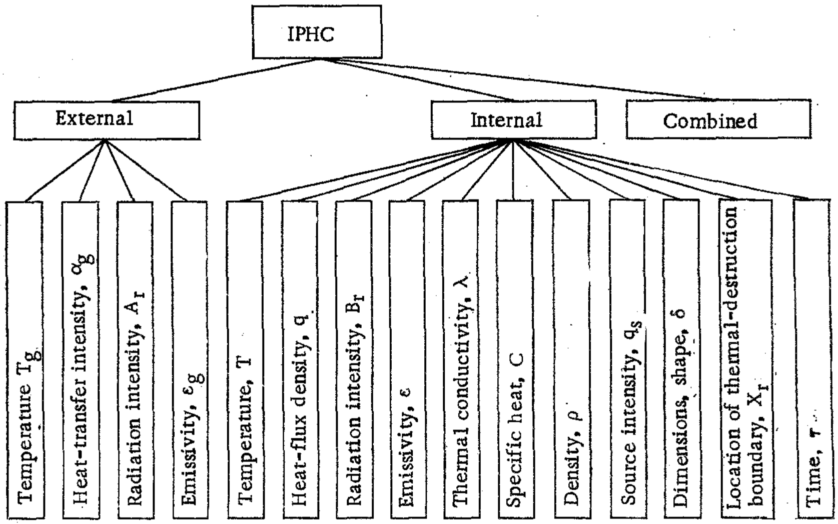

Fig. 1. Classification of inverse problems of heat conduction (IPHC).

Let us examine the non-steady-state heat mode in a flat wall under nonsymmetric heat-transfer conditions. Let a flat wall of thickness δ be given. The thermophysical properties of the wall material are characterized by the quantities λ, c, ρ, and α. The temperature at all points of the wall is constant and equal to T_i at the initial instant. On one side a hot medium with temperature T_g flows over the wall, and a cold medium with a temperature T_c on the other. The heat transfer between the wall and the hot medium is characterized by the coefficient of heat transfer α_g and between the wall and the cold medium by α_c. In the general case, the heat-transfer coefficients α_g and α_c are not equivalent. The problem is formulated mathematically as follows for a one-dimensional field:

$$
\begin{gathered}
c\rho\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right);\\
T=T_i;\\
\lambda\frac{\partial T}{\partial x}+\alpha_g(T_g-T)=0 \quad \text{or}\\
\lambda\frac{\partial T}{\partial x}+\alpha_k(T_g-T)+A_r\left[\left(\frac{T_g}{100}\right)^4-B_r\left(\frac{T}{100}\right)^4\right]=0;\\
\lambda\frac{\partial T}{\partial x}-\alpha_c(T_c-T)=0.
\end{gathered}
\tag{1}
$$

In this formulation the solution of the inverse problems of heat conduction reduces to the determination of T_s, c, ρ, λ, $q_s=-\lambda(\partial T/\partial x)_s$, B_r, ε, and δ for the internal, and T_g, α_g, A_r, and ε_g for the external problems.

Besides the analytical and numerical methods using a regularizing algorithm and an electronic digital computer, the method of analog electrical simulation on specialized analog computers is well recommended for the solution of inverse problems of heat conduction. Characteristic of it are the quite fast response, the physical graphic solution, the possibility of intervention during the solution and the rapid verification of the results obtained by solving the direct problem on the same model, and the great simplicity and high accuracy. Underlying the method of electrical simulation is the strict mathematical analogy between the processes in the original and in the model. By using the scheme of replacing the heat-conducting medium successively by connected electrical cells of resistors and capacitances, we obtain a model in which the transfer electrical process is represented by the system of equations

$$
\begin{gathered}
c_e\frac{\partial u}{\partial \tau_e}=\frac{\partial}{\partial x_e}\left(\frac{1}{r}\cdot\frac{\partial u}{\partial x_e}\right);\quad u=u_i;\\
\frac{\partial u}{\partial x_e}+\frac{r}{R_g}(u_g-u)=0;\\
\frac{\partial u}{\partial x_e}-\frac{r}{R_c}(u_c-u)=0.
\end{gathered}
\tag{2}
$$

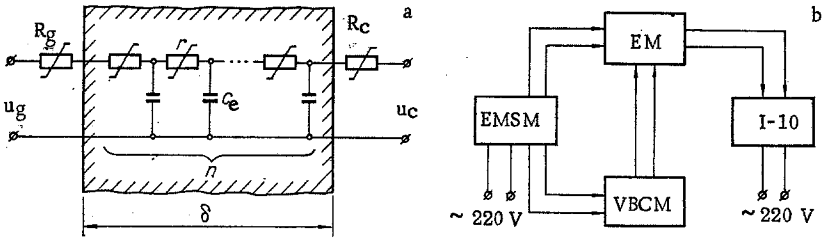

Fig. 2. Schematic diagram of the electrical model (a) and block diagram of the simulating unit (b) (EMSM, electrical model supply module; VBCM, variable boundary-condition module; EM, field of the electrical model; I-10, display).

The diagram of the electrical model, in principle, as well as the block diagram of the modeling apparatus, are shown in Fig. 2. It should be noted that it is expedient to use cathode displays of the I-6 or I-10 type or cathode-ray oscilloscopes of the C 1-4, C 1-49, C 1-68 types as being more graphic recorders in solving inverse problems. Following [1], we have

$$c_e = \frac{k_l^2}{ark_\tau}; \quad k_l = \frac{\delta}{n};$$
$$R_g = \frac{\lambda}{\alpha_g \delta} rn; \quad k_\tau = \frac{\tau}{\tau_e}; \tag{3}$$
$$\alpha_g R_g = \alpha_c R_c; \quad k_T = \frac{T}{u}.$$

to determine the regulatable parameters of the electrical model during simulation.

The solution of direct and inverse problems of heat conduction on specialized electrical models consists in the following. The system of equations with the uniqueness conditions (1), which describes heat transfer to the solid, is known in the solution of direct problems of heat conduction. This system of equations permits determination of all the parameters of the electric model (3) such as the resistance, capacitance, and quantity of cells r, c_e, and n; the boundary resistors R_g and R_c the coordinate k_l, time k_τ, and temperature k_T scales, and the initial voltage distribution by means of known relationships. In other words, knowledge of the boundary conditions and thermophysical parameters (coefficients) of the energy equation permits complete computation of the regulatable parameters of the model. The process of solving the problem consists in recording the voltage change at a desired point in time. As a rule, the aim in solving the inverse problem of heat conduction is to determine the boundary conditions (T_g, α_g, T_c, α_c, etc.) or the thermophysical parameters (λ, c, ρ, etc.). This means that one or more of the electrical model parameters (u_g, R_g, u_c, R_c, r, c_e, etc.) are unknown and to be determined. The solution is carried out on the electrical model by selecting the desired quantity. Agreement between the given and measured temperature curve at a fixed point of the solid is a criterion of the estimate. Diagrams for the solution of the direct and inverse problems of heat conduction and its simulation are represented in Fig. 3, where the direction of inserting the data and the path of the solution is shown by arrows.

To illustrate the method, let us present an example of solving the inverse problem of heat conduction. Let a flat wall be given at some point of which the temperature is measured. The temperature of the hot medium flowing over the left surface of the wall and the nature of its variation must be found according to the data of temperature measurements in the wall.

The solution of this problem for a steel wall was carried out on an electrical model of 20 cells rc_e (r = 0-3.2 kΩ, c_e = 100 μF.

A number of problems was solved (Fig. 4) to clarify the influence of a brief change in temperature of the medium T_g on the nature of the behavior of the temperature field in the wall. The temperature of the medium in the first 20% of the time of the thermal effect hence varied according to different laws (Fig. 4a), then remained constant to the end of the thermal effect.

It follows from Fig. 4b that brief changes in the temperature of the medium T_g damp out rapidly with removal from the surface, especially in the case of boundary conditions of the third kind. This is explained

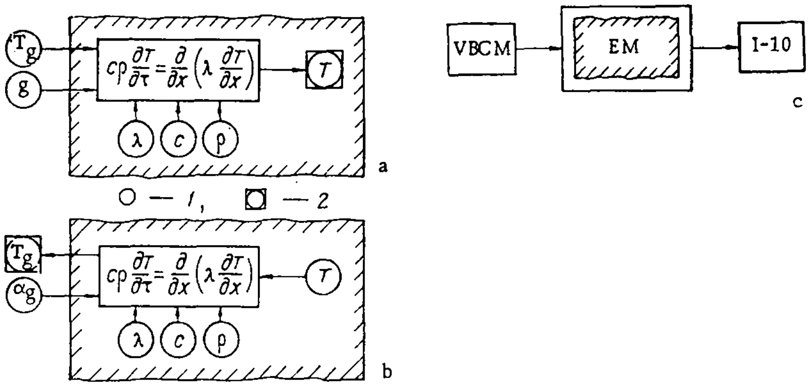

Fig. 3. Diagram for the solution of direct (a) and inverse (b) problems of heat conduction and the electrical simulation (c) of these problems (1) input data; 2) desired quantity; VBCM, variable boundary-condition module; EM, field of the electrical model; I-10, display).

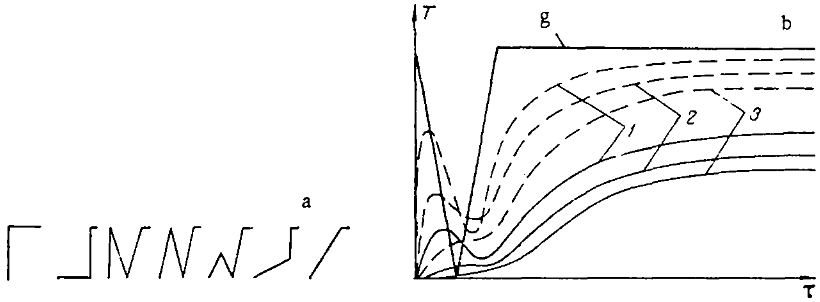

Fig. 4. Nature of the change in temperature of the medium (a) and change in the wall temperature (b) at different distances from the heated (left) surface (1) $x/\delta = 0.25$; 2) 0.5; 3) 1.0 for boundary conditions of the first (dashed line) and third (solid line) kinds.

by the presence of thermal resistances on both the boundary and within the solid. The desired changes in the temperature of the medium not only diminish in amplitude but are also deformed with time. Hence, the nature of the change in the desired quantity should be established primarily in solving inverse problems. This can evidently be done if the source of the initial information were located in the zone of sensitivity of the noted oscillations.

We understand the zone of sensitivity to be the space in which the primary information source directly or indirectly reacts on the fundamental changes of the desired quantity.

The investigations executed show that the zone of sensitivity diminishes with the increase in the error of the primary information source, with the increase in the frequency, and with the diminution in the amplitude of the oscillations in the desired quantity. The choice of the location of the information source influences the nature and accuracy of the results. However, it should be noted that if a search of the mean values is made, then the zone of sensitivity is substantially expanded.

Therefore, it should be seen in solving inverse problems that the information source is in the zone of sensitivity and only then should one proceed to the solution.

In the example considered, the reproduced temperature of the medium differs from the true value by not more than 5% when the information source is located in the zone of sensitivity.

NOTATION

Thermal process: T, T_s, T_i, T_g, T_c, wall, surface, initial, gas stream and environmental temperatures, respectively; q_s, heat-flux density on the surface; λ, c, a, thermal-conductivity, specific-heat, and thermal-diffusivity coefficients; ρ, density; α_k, convective heat-transfer coefficient; α_g, α_c, total heat-transfer coefficient of the wall from the hot and cold medium; A_r, B_r, radiation coefficients; ε, ε_g, emissivity of the wall and the gas flow; δ, wall thickness; x, coordinate; τ, time. Electrical process: u, u_i, u_g, u_b, cell, initial, left- and right-side model boundary voltages, respectively; r, c_e, ohmic resistance and capacitance of the model cell; R_g, R_c, left- and right-side model boundary resistances; n, number of model cells; k_l, k_τ, k_T, coordinate, time, and temperature scales; x_e, cell coordinate; τ_e, time.

LITERATURE CITED

1. M. P. Kuz'min, Electrical Simulation of Nonstationary Heat Transfer Processes [in Russian], Énergiya, Moscow (1974).

USE OF A HYPERBOLIC EQUATION IN THERMAL-CONDUCTIVITY THEORY

V. A. Bubnov and I. A. Solov'ev

UDC 536.33

A solution of the telegraph equation is given which is close to a self-similar solution.

1. Singularities in Solutions of the Classical Equation of Thermal Conductivity. In the simulation of thermal processes by the equation of thermal conductivity,

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}, \tag{1}$$

certain singularities occur. Actually, we consider the fundamental solution of Eq. (1)

$$T_0(x, t) = 1/\sqrt{4\pi at} \exp[-x^2/(4at)] \tag{2}$$

and find the mean value of the square of the temperature displacement from its initial position during the time t:

$$\overline{\Delta x^2} = \int (x - x_0)^2 T_0(x, t)\, dx / \int T_0(x, t)\, dx = 2at. \tag{3}$$

We define the rate of temperature displacement in the following manner:

$$V = \frac{d}{dt}\left(\sqrt{\overline{\Delta x^2}}\right) = \sqrt{a/(2t)}. \tag{4}$$

It then follows that the temperature nonuniformity is propagated instantaneously at the initial time. A similar paradox occurs in the theory of Brownian motion [1].

Using the fundamental solution, we find an equation for the surface of maximum temperature. To do this, we differentiate Eq. (2) with respect to time and equate the result to zero. Then $x^2 - 2at = 0$, hence $x = \sqrt{2at}$ and

$$V_{\max} = \frac{dx}{dt} = \sqrt{a/(2t)}, \tag{5}$$

i.e., the expression for the rate of displacement of the surface of maximum temperature agrees with Eq. (4) and $V_{\max}$ has a marked singularity.

The use of the classical equation of thermal conductivity in phase-transition problems also leads to a similar paradox. Actually, in the well-known Stefan solution [2], the law of motion for the freezing line has the form $z = p\sqrt{t}$ so that

$$V_z = \frac{dz}{dt} = p/(2\sqrt{t}).$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1131-1135, December, 1977. Original article submitted May 18, 1976.

 0022-0841/77/3306-1512$07.50